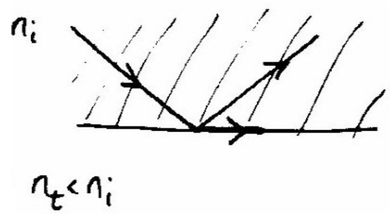
θ_c - critical angle for total int. reflection



$$n_i \sin\theta_c = n_t$$

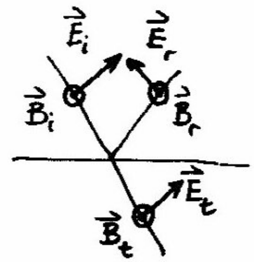
$$\sin\theta_c = \frac{n_t}{n_i} = n_{ti}$$

for $\theta_i > \theta_c$ treatment is not quite right (evanescent waves)

in general, $t_\perp - r_\perp = 1$.

<u>Case (ii)</u> $\vec{E}$ // to plane of incidence

$\Rightarrow \vec{B} \perp$ " " " "



ME $\Rightarrow$ $E_{oi}\cos\theta_i - E_{or}\cos\theta_r = E_{ot}\cos\theta_t$

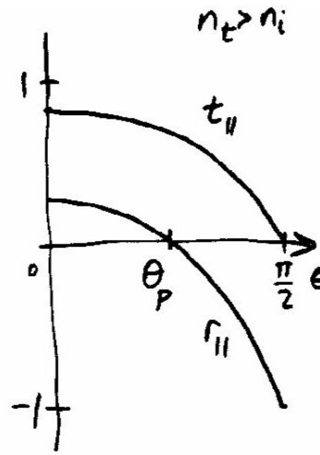
$$n_i E_{oi} + n_r E_{or} = n_t E_{ot}$$

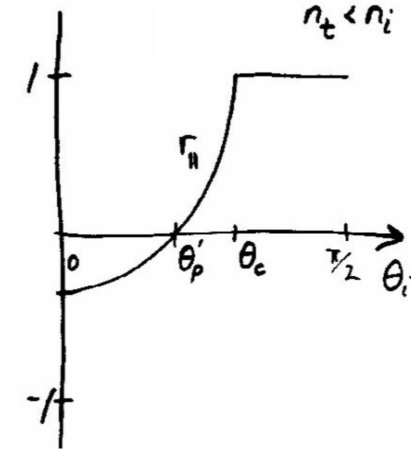
combine: $(\theta_r = \theta_i)$

$$\left(\frac{E_{or}}{E_{oi}}\right)_{\parallel} \equiv r_{\parallel} = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_i\cos\theta_t + n_t\cos\theta_i}$$

$$\left(\frac{E_{ot}}{E_{oi}}\right)_{\parallel} \equiv t_{\parallel} = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i}$$

(r's and t's can be further simplified using Snell's law).



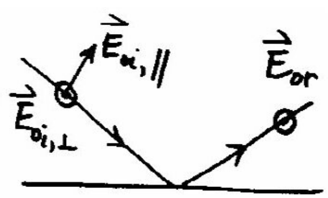


<u>note:</u> $t_\parallel - r_\parallel \neq 1$, but $t_\parallel + r_\parallel = 1$ at normal incidence

again, for $\theta > \theta_c$ treatment not quite right.

<u>look at θ_P</u> "Brewster's angle"

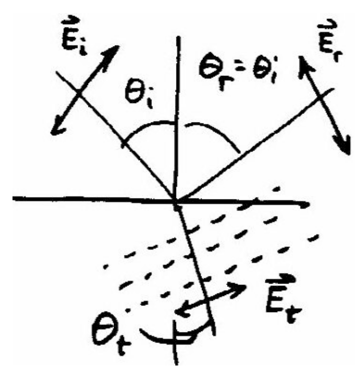
mixed polarized incident:



output only $\perp$ polarization

why?

oscillators don't emit $\perp$ to their oscillation direction:



if $\theta_t + \theta_r = \pi/2$ then $E_r = 0$.

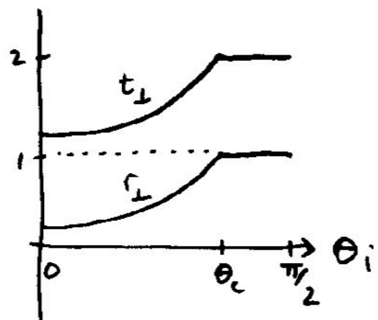
$$\theta_t = \pi/2 - \theta_i$$

$$n_i \sin\theta_p = n_t \sin\theta_t = n_t\cos\theta_p$$

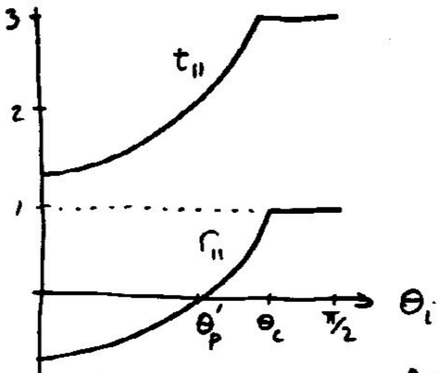
$$\Rightarrow \boxed{\tan\theta_p = \frac{n_t}{n_i}}$$

similarly for θ_p'

return to examine $t_\perp, t_\parallel$ for $n_t < n_i$:



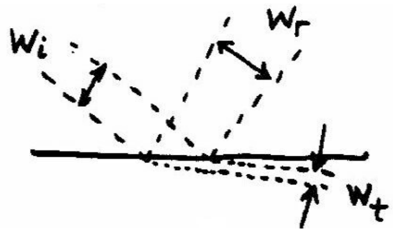
$(t_\perp = 1 + r_\perp)$



$(t_\parallel = 1 - r_\parallel \ ; \ \theta_i = 0^\circ)$

- find large t values! Does this violate conservation of energy?

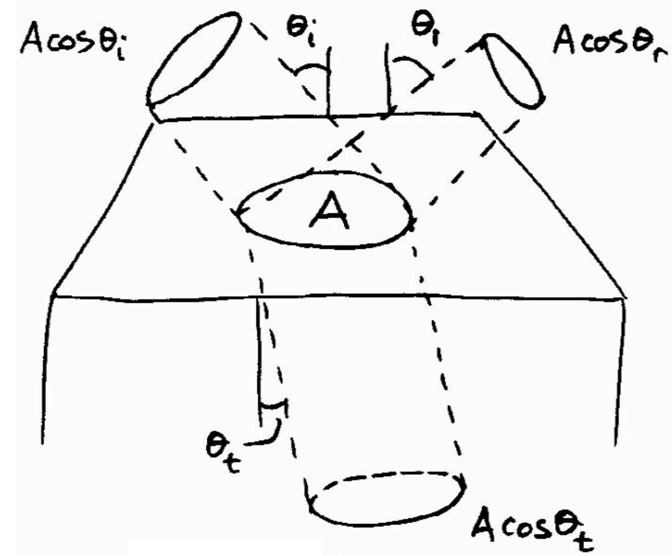
No! cross-sectional area of beams also varies!



$w_t \to 0$ for $\theta_t \to \pi/2$

$\Rightarrow$ no energy in beam.

in detail:



I_i, I_r, I_t flux densities (intensities) (W/m^2)

incident power $= I_i A\cos\theta_i$

reflected " $= I_r A\cos\theta_r$

transmitted " $= I_t A\cos\theta_t$

then,

$$\text{reflectance } R \equiv \frac{I_r A\cos\theta_r}{I_i A\cos\theta_i} = \frac{I_r}{I_i}$$

$$\text{transmittance } T \equiv \frac{I_t A\cos\theta_t}{I_i A\cos\theta_i} = \frac{I_t \cos\theta_t}{I_i \cos\theta_i}$$

relate using $I_i A\cos\theta_i = I_r A\cos\theta_r + I_t A\cos\theta_t$

and $I = \frac{v\varepsilon}{2} E_0^2 = \frac{cn^2\varepsilon_0}{2n} E_0^2 \propto nE_0$

$\Rightarrow n_i E_{0i}^2 \cos\theta_i = n_i E_{0r}^2 \cos\theta_i + n_t E_{0t}^2 \cos\theta_t$

$$1 = \left(\frac{E_{0r}}{E_{0i}}\right)^2 + \frac{n_t \cos\theta_t}{n_i \cos\theta_i}\left(\frac{E_{0t}}{E_{0i}}\right)^2$$

or $\boxed{1 = R + T}$

also, can write

$$R = \left(\frac{E_{0r}}{E_{0i}}\right)^2 = r^2$$

$$T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i}\left(\frac{E_{0t}}{E_{0i}}\right)^2 = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t^2$$

and $R_\perp \equiv r_\perp^2$, $R_\parallel \equiv r_\parallel^2$

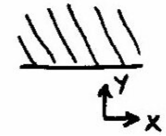
$$T_\perp \equiv \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t_\perp^2 , \quad T_\parallel \equiv \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t_\parallel^2$$

and find

$$R_\parallel + T_\parallel = 1 , \quad R_\perp + T_\perp = 1$$

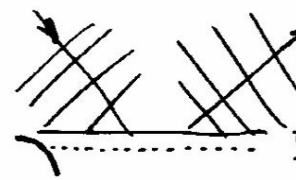
Evanescent Waves ($n_t < n_i$, $\theta_i > \theta_c$)

- impossible to satisfy boundary conditions with real values of k_y ;



need $k_y = i\beta$,

wave decays exponentially,



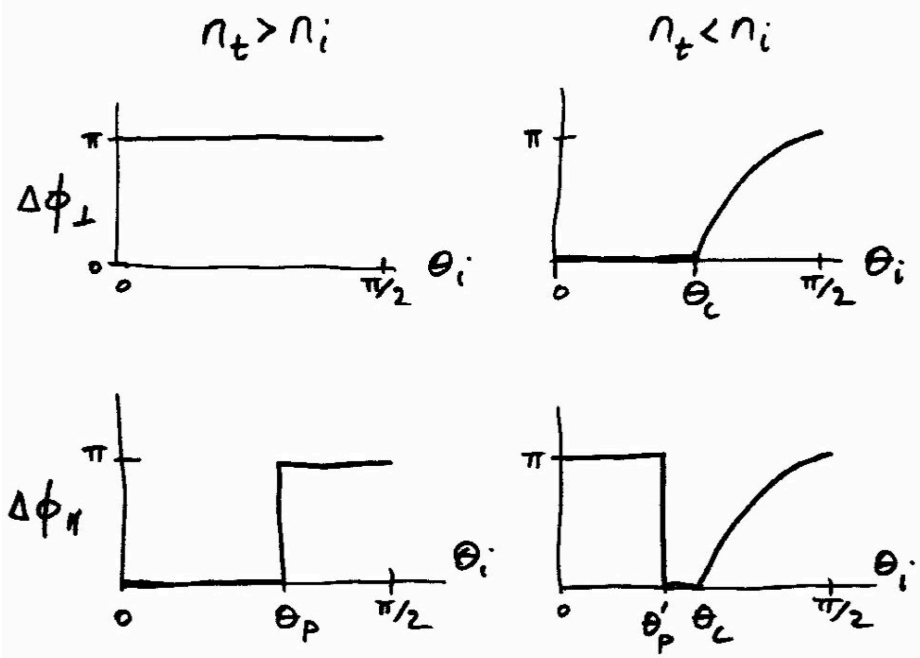
turns out, $\beta = \left(\frac{\sin^2\theta_i}{n_{ti}^2} - 1\right)^{1/2} \to 0$ as $\theta \to \theta_c$

For incident wave of infinite extent,

$w_i \to \infty$, then fraction of power in evanescent wave is zero.

(for finite w_i, then must consider diffraction at edges of incident wave).

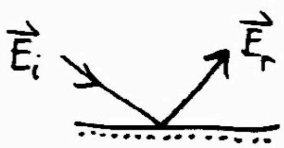
Phase Shift of Reflected Wave:



- very important for later applications of eg. multiple reflections in thin films,

- application of Fresnel Eqns:

Ellipsometry



- measure polarization and amp. of reflected beam, vs. ω and/or vs. θ_i. Determine $\tilde{n}$ for material and for thin films on material.